

Large Mixing Angle Sterile Neutrinos and Pulsar Velocities

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We investigate the momentum given to a protoneutron star, the pulsar kick, during the first 10 seconds after temperature equilibrium is reached. Using a model with two sterile neutrinos obtained by fits to the MiniBooNE and LSND experiments, which is consistent with a new global fit, there is a large mixing angle, and the effective volume for emission is calculated. Using formulations with neutrinos created by URCA processes in a strong magnetic field, so the lowest Landau level has a sizable probability, we find that with known parameters the asymmetric sterile neutrino emissivity might account for large pulsar kicks.

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I. INTRODUCTION

The existence of sterile neutrinos, in addition to three active neutrinos of the standard model, is of great interest for both particle physics and astrophysics. The Los Alamos LSND experiment found evidence for antineutrino oscillation[1]. An analysis of LSND, along with other short-baseline experiments, found[2] that the standard model is not consistent with the data and the best model has two sterile neutrinos in addition to three active neutrinos.

The recent experiment by the MiniBooNE Collaboration[3] found that the data for electron neutrino appearance showed an excess at low energies, in comparison to what was expected in the standard model. This data, along with the LSND data, has been analyzed in a model with two light sterile neutrinos[4], and compared to MiniBooNE data[5]. The mixing angles of two light sterile neutrinos were extracted. (See, however, Ref.[6], which questions the accuracy of the results of Ref[4]). An analysis of MiniBooNE, LSND, and other experiments[7], based in part on earlier work with the seesaw mechanism[8], also found that the preferred model to fit the data consists of two light sterile neutrinos, in addition to three active neutrinos.

Very recently a global analysis of updated MiniBooNE data, along with data from LSND, KARMEN, NOMAN, Bugey, CHOOZE, CCFR84, and CDHS has been carried out[9]. There are two possible scenarios, both compatible with the results of

Refs.[4, 5]. Fits to appearance and disappearance with a 3+1 hypothesis have very good χ^2 probabilities, and also find CPT violation. The CPT violation, with different mixing angles and mass differences for neutrinos vs antineutrinos, might be understood as matter effects, similar to the MSW effect[10, 11], but this has not yet been proved. Also a 3+2 model was shown to give a good χ^2 probability and is compatible within the range of the parameters for the 3+2 model used in Refs.[4, 5], but without the uncertainty pointed out in Ref.[6].

In the present work we apply this range of fits[4, 5] to MiniBooNE/LSND to the study of pulsar velocities. The gravitational collapse of a massive star often leads to the formation of a neutron star, a pulsar. It has been observed that many pulsars move with linear velocities of 1000 km/s or greater, called pulsar kicks. See Ref.[12] for a review. Due to the high density, the active neutrinos have a small mean free path, and only escape at the surface of the neutrinosphere. During the first ten seconds after temperature equilibrium is attained during the gravitational collapse of a massive star, the main cooling is via emission of neutrinos produced by the URCA process, and during the next ten seconds via neutrinos produced by the modified URCA process. We have investigated the pulsar kicks which arise from the modified URCA processes in the time interval 10-20 sec after the supernova collapses, when the neutrinosphere is just inside the protoneutron star. With a strong magnetic field and temperature, so that the population of the lowest Landau level is approximately 0.4 of the total occupation probability, and we find large pulsar kicks[13].

The largest neutrino emission after the supernova collapse takes place during the first 10 sec-

onds, with URCA processes dominant. The possibility of pulsar kicks from anisotropic neutrino emission due to strong magnetic fields during this time was discussed more than two decades ago[14]. It has been shown[15] that, with the strength of the magnetic field expected during this period, the lowest Landau level has a sizable occupation probability, which produces the neutrino emission asymmetry that is needed for pulsar kicks. However, due to the high opacity for standard model neutrinos in the dense region within the neutrinosphere, few neutrinos are emitted, and the large pulsar kick is not obtained[16].

Sterile neutrinos with a small mixing angle have small opacities. It has been shown[17] that, using the model of Ref.[15] and assuming the existence of a heavy sterile neutrino (mass > 1 keV) with a very small mixing angle constrained to fit dark matter, the pulsar kicks could be explained. More recently the effects of such sterile neutrinos with large masses and small mixing angles have been studied for other processes[18]. See Ref.[19] for a review of processes that might be associated with dark matter sterile neutrinos.

In the present paper we use the fits of Refs.[4, 5] with two sterile neutrinos to investigate the possibility of obtaining the large pulsar velocities which have been observed. The values of the masses and mixing angles are within the range found in calculations based on the seesaw mechanism[7]. Note that our model differs from that of Ref[17] in that with a much larger mixing angle there is a higher probability of sterile neutrinos, but a much smaller effective volume, due to a larger opacity. However, as we shall show, since the mean free path is much larger than those of standard neutrinos, under the conditions in which standard neutrinos produce a pulsar velocity of 2-300 km/s, the MiniBoone/LSND sterile neutrinos can give a kick of more than 1000 km/s.

II. ASYMMETRIC STERILE NEUTRINO EMISSIVITY AND PULSAR KICKS IN LIGHT TWO-STERILE NEUTRINO MODEL

Within about 1 second after the gravitational collapse of a large star, the neutrinosphere is formed with a radius of about 40 km, with temperature equilibrium. Within about 10 seconds about 98% of neutrino emission occurs, with neutrinos produced mainly by URCA processes. Due to the strong magnetic field, neutrino momentum asymmetry is produced within the neutrinosphere, but with a small

mean free path they are emitted only from a small surface layer of the neutrinosphere, and the pulsar kick cannot be accounted for. If a standard active neutrino, say the electron neutrino, oscillates into a sterile neutrino, it will escape from the protoneutrino star and neutrinosphere, unless it oscillates back into the active neutrino. The mixing angle plays a key role. In the work of Fuller et al[17] the mixing angle is so small that the sterile neutrinos are emitted. In the present work the starting point is the analysis of MiniBoone and LSND data, with the two or more sterile neutrinos with small masses and large mixing angles. Before we can proceed, however, it is essential to determine possible effects of the high density and temperature of the medium on the mixing angles.

A. Mixing Angle in Neutrinosphere Matter

It has long been known that dense matter can affect neutrino states. The famous MSW effect[10, 11] for understanding solar neutrinos, and the study of oscillations of high energy neutrinos[20] are studies of mixing of active neutrinos in matter. There have been many other studies. In the present work we are dealing with sterile/active neutrino mixing given by the mixing angle θ_m in neutrinosphere matter

$$\begin{aligned} |\nu_1\rangle &= \cos\theta_m |\nu_e\rangle - \sin\theta_m |\nu_s\rangle \\ |\nu_2\rangle &= \sin\theta_m |\nu_e\rangle + \cos\theta_m |\nu_s\rangle \end{aligned} \quad (1)$$

In the work Ref[17] it was shown that the mixing angle for sterile neutrinos that can account for dark matter as well as those produced in the neutron star core is almost the same as the vacuum value. Starting from the much larger mixing angles for the sterile neutrinos that seem to account for the MiniBoone, LSND data, we need the value of the mixing angles in the neutrinosphere, as we discuss below. The effective mixing angle in matter, θ_m can be related to the vacuum mixing angle, θ by[21]

$$\sin^2(2\theta_m) = \frac{\sin^2(2\theta)}{\sin^2(2\theta) + (\cos(2\theta) - \frac{2pV^T}{(\delta m)^2})^2}. \quad (2)$$

In Eq(2) V^T is the finite temperature potential, while the finite density potential due to asymmetries in weakly interacting particles has been dropped as it vanishes when temperature equilibrium is reached[21]. A convenient form for V^T , with

the background of both neutrinos and electrons included, is given in Ref[22]

$$V^T = \frac{28\pi G_F^2}{45\alpha} \sin^2\theta_W (1 + 0.5\cos^2\theta_W) pT^4, \quad (3)$$

with G_F, θ_W the standard weak interaction parameters and $\alpha = 1/137$. Assuming $T=20a$ MeV, with $a \leq 1.0$, $p=b$ MeV, and $(\delta m)^2 = 1.0 \text{ eV}^2$ [4, 6], we find

$$\frac{2pV^T}{(\delta m)^2} \simeq 5.1 \times 10^{-3} b^2 a^4 \ll \cos(2\theta). \quad (4)$$

Therefore, the mixing angle in the neutrinosphere medium is approximately the same as the vacuum mixing angle. This agrees with Ref[17].

B. Emissivity With a Light Sterile Neutrino

We now use the fits to MiniBooNe and LSND with light sterile neutrinos to estimate pulsar kicks. The MiniBooNE results are consistent with the LSND results and CPT only if there are at least two sterile neutrinos. Models with three sterile neutrinos have also been considered[6, 8]. Fits to the MiniBooNE experiment and the LSND results by Ref.[4] in Ref.[5] with two sterile neutrinos are shown in Fig. 1.

From the Sterbenz/Maltoni-Schwetz fits one finds for the mixing angles of the two sterile neutrinos:

$$\begin{aligned} (\sin 2\theta_{1s})^2 &= 0.004 \\ (\sin 2\theta_{2s})^2 &= 0.2, \end{aligned} \quad (5)$$

and the masses are negligibly small. Note that this is in contrast to the parameters of Ref.[17], with the constraint of dark matter giving a mixing angle of $(\sin 2\theta_{dm})^2 \simeq 10^{-8}$, and a mass greater than 1 keV. In our present work we will use values for $(\sin 2\theta)^2$ in the range 0.2 to 0.004 to estimate the pulsar kick, which is compatible with the recent global analysis[9]

The probability of asymmetric emission, giving a pulsar kick, does not depend directly on the sterile neutrino mass in our model, but is proportional to the $(\sin 2\theta_s)^2$. It is the large mixing angles found in fits to MiniBooNe and LSND that lead us to carry out the investigation in the present paper.

MiniBooNe, LSND Data and Two Sterile Neutrinos
M. Maltoni and T. Schwetz, hep-ph/0705.0107 (2007)

Figure by M. Sterbenz, LANL

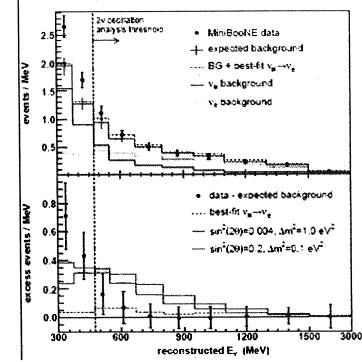


FIG. 1: $\sin^2(2\theta_{1s})=0.004$; $\sin^2(2\theta_{2s})=0.2$

C. General Formulation of Neutrino Emissivity With a Strong Magnetic Field

The neutrino emissivity is given in general form in many papers, e.g., see Refs [23, 24]:

$$\begin{aligned} e^\nu &= \Pi_{i=1}^4 \int \frac{d^3 p^i}{(2\pi)^3} \frac{d^3 q^\nu}{2\omega^\nu (2\pi)^3} \int \frac{d^3 q^e}{(2\pi)^3} \\ &\quad (2\pi)^4 \sum_{s_i, s^\nu} \frac{1}{2\omega_L^e} \omega^\nu \mathcal{F} M^\dagger M \\ &\quad \delta(E_{final} - E_{initial}) \delta(\vec{p}_{final} - \vec{p}_{initial}), \end{aligned} \quad (6)$$

where M is the matrix element for the URCA process and \mathcal{F} is the product of the initial and final Fermi-Dirac functions corresponding to the temperature and density of the medium. The main source of the asymmetric emissivity that produces the pulsar velocity is the fact that the electron has a large probability to be in the lowest ($n=0$) Landau level.

See Refs [25, 26] for a discussion of Landau levels. The asymmetric emissivity can be seen by considering the weak axial interaction, W_A ,

$$W_A = -\frac{G}{\sqrt{2}}g_A\chi_p^\dagger\vec{l}\cdot\vec{\sigma}\chi_n \quad (7)$$

$$l_\mu = \bar{\Psi}(q^e)\gamma_\mu(1-\gamma_5)\Psi(q^\nu),$$

with $G = \frac{10^{-5}}{m_n^2}$, $g_A = 1.26$, the χ are the nucleon spinors, and the lepton wave functions are $\Psi(q^e), \Psi(q^\nu)$, where q^e and q^ν are the electron and antineutrino momenta, respectively. The key to the asymmetric emission is given by the trace over the leptonic currents, $Tr[l_i^\dagger l_j]$,

$$\int d^2q_\perp^e Tr[l_i^\dagger l_j] \simeq 8\pi E^e [(q^\nu)^j \delta_{i3} + (q^\nu)^i \delta_{j3} - \delta_{ij} (q^\nu)^3] (\hat{q}^e = \hat{B} = \hat{z}), \quad (8)$$

with the magnetic field B in the z direction. We only consider the weak axial force, which is dominant. Using the relationship given in Eq(8) one can show that the result of the traces and integrals over the axial product matrix element has the form ($\hat{B} = \hat{z}$)

$$\int d^2q_\perp^e |M_A|^2 \propto (q^\nu)^z. \quad (9)$$

Details are given in Ref [13], where it is shown that the asymmetric neutrino emissivity, using the general formulation of Ref [23], is

$$\epsilon^{AS} \simeq 0.64 \times 10^{21} T_9^7 P(0) \times f$$

$$\text{erg cm}^{-3} \text{ s}^{-1} = p_{ns} c \frac{1}{V_{eff} \Delta t}, \quad (10)$$

where $T_9 = T/(10^9 K)$, p_{ns} is the neutron star momentum, $P(0)$ is the probability of the electron produced with the antineutrino being in the lowest Landau state, $f=0.52$ is the probability of the neutrino being at the $+z$ neutrinosphere surface [13], V_{eff} is the volume at the surface of the neutrinosphere from which neutrinos are emitted, and $\Delta t \simeq 10s$ is the time interval for the emission.

We derive $P(0)$ and V_{eff} , the effective volume for the emissivity, in the next two subsections.

D. $P(0)$ = Probability for the Electron to be in the $n=0$ Landau Level

Just as in our previous work in which Landau levels play a crucial role[13], only the lowest Landau level, for which the helicity is $-1/2$ (rather than

$\pm 1/2$ as with the usual Dirac spinors) gives asymmetric emission. The probability that an electron in a strong magnetic field is in the lowest ($n=0$) Landau level, $P(0)$, can be calculated from the temperature, T , and the energy spectrum of Landau levels[25, 26]. A particle with momentum p and effective mass m_e^* in a magnetic field B in the n th Landau level has the energy

$$E^L(p, n) = \sqrt{p^2 + (m_e^*)^2 + 2(m_e^*)^2 \frac{B}{B_c} n}, \quad (11)$$

with $B_c = 4 \times 10^{13} \text{ G}$, and m_e^* is the effective mass of the electron at the high density of the protoneutron star and neutrinosphere.

From standard thermodynamics the probability of occupation of the $n=0$ Landau level, $P(0)$, is given by[17]:

$$P(0) = \frac{F(0)}{F(0) + 2 \sum_{n=1}^{\infty} F(n)}, \quad (12)$$

where $F(n)$, with magnetic field B , temperature T , and chemical potential μ , is

$$F(n) = \int_{p_{min}}^{\infty} dp \frac{[m_n - m_p - E^L(p, n)]^2}{1 + \exp[(E^L(p, n) - \mu)/T]} \quad (13)$$

The electron energy is restricted to magnitudes greater than μ , but the integrals in Eq.(7) are insensitive to p_{min} , so we take $p_{min}=0$ as in Ref[17].

We agree with the estimate of Ref.[17] for $P(0)$. Note that if we had used the free electron mass, m_e , in the Landau energies (Eq.(5)) we would have obtained a much smaller value for $P(0)$. For $B = 10^{16} \text{ G}$, $\mu = 40 \text{ MeV}$, $m_e^* = 4 \text{ MeV}$ and $T_{\nu-sphere} = 20 \text{ MeV}$, $P(0) \simeq 0.3$. This is similar to our estimate of $P(n=0) \simeq 0.4$ at the surface of the protoneutron star at about 10 seconds[13]. Therefore our result for asymmetric emissivity differs from that of Fuller et al [17] mainly in that we have a much larger mixing angle, and a much smaller effective volume, since the sterile neutrinos oscillate back to active neutrinos within the neutrinosphere; and therefore our emission only takes place near the surface of the neutrinosphere. However, in contrast with purely active neutrino emission in which the opacity results in very small pulsar kicks [16], the sterile neutrinos have a much larger effective volume, and can therefore produce much larger pulsar velocities

E. Estimate of V_{eff} = Effective Volume for Emission

To estimate V_{eff} we make use of the early study of opacity in about the first 20s of the creation of a neutron star via a supernova collapse [27, 29, 30], and a recent detailed study of neutrino mean free paths[28]. Since the mean free path of the sterile neutrino is determined by that of the standard neutrino to which it oscillates, λ , we make use of studies of active neutrino mean free paths. First note that the neutrino mean free path is given by

$$1/\lambda = \int \frac{d^3p}{(2\pi)^3} M_{fi} [1 - n(q)] \times (1 + e^{(\mu_\nu - E_\nu)/kT}), \quad (14)$$

where M_{fi} is the weak matrix element and $n(q)$ is the Fermi distribution. For the calculation of the sterile neutrino, for which $M_{fi} = 0$, one can use the value of $1/\lambda$ with a factor of $\sin^2(2\theta)$ from the matrix element and another such factor from the occupation probability. From the results of previous authors, for T in the 10 to 20 Mev range and μ in the 20 to 40MeV range, we estimate that $\lambda \simeq 1.0\text{cm}$. This gives a range for the effective sterile neutrino mean free path

$$\lambda_s \simeq 5.0 \text{ to } 250 \text{ cm}. \quad (15)$$

For a neutrinosphere radius of 40 km, with $\lambda_s \ll R_\nu$ this gives us $V_{eff} = (4\pi/3)(R_\nu^3 - (R_\nu - \lambda_s)^3) \simeq 4\pi R_\nu^2 \lambda_s$.

From Eq.(10), $R_\nu = 40 \text{ km}$, and $\lambda=1.0 \text{ cm}$,

$$p_{ns} = M_{ns} v_{ns} \simeq \frac{0.67 \times 10^{25}}{\sin^2(2\theta)} T_9^7 \text{ gm} \frac{\text{cm}}{\text{s}}, \quad (16)$$

with $T_9 = \frac{T}{10^9 \text{ K}}$. Taking the mass of the neutron star to equal the mass of our sun, $M_{ns} = 2 \times 10^{33} \text{ gm}$, we obtain for the velocity of the neutron star

$$v_{ns} \simeq 3.35 \times 10^{-7} \left(\frac{T}{10^{10} \text{ K}} \right)^7 \frac{1}{\sin^2(2\theta)} \frac{\text{km}}{\text{s}}, \quad (17)$$

For example, for $T=10 \text{ Mev} = 1.16 \times 10^{11} \text{ kK}$,

$$v_{ns} \simeq 47.3 \frac{\text{km}}{\text{s}} \text{ to } 2,370 \frac{\text{km}}{\text{s}}, \quad (18)$$

which means that sterile neutrino emission could account for the large pulsar kick with the parameters extracted from Refs [4, 5]. If we use the physical parameters that give Eq.(18) for electron neutrinos,

we obtain a pulsar velocity of $v_{ns} = 95 \text{ km/s}$, which is consistent with previous predictions by several authors.

It should be noted that the study of the Mini-BooNE and LSND results are in progress, and the mixing angles that result could be different from those obtained in Refs.[4, 5]. For this reason we have used a range of parameters. Preliminary data from the MiniBooNE/Minos experiment[31], however, is consistent with the MiniBooNE results[3]. We also once more point out that although Ref [6] questions the accuracy of the parameters extracted by Ref [4], our model is compatible with the recent global analysis[9], with very good χ^2 probabilities.

III. CONCLUSIONS

Because of the strong magnetic fields in protoneutron stars and the associated neutrinosphere, the electrons produced in the URCA processes that dominate neutrino production in the first 10 seconds have a sizable probability, $P(0)$, to be in the lowest ($n=0$) Landau level. This leads to asymmetric neutrino momentum. With the mixing angles found in Refs[4, 5], we find that the sterile neutrinos produced during this period for high luminosity pulsars can give the pulsar velocities of greater than 1000 km/s, as observed. We emphasize that our results are consistent sterile neutrino parameters based on a global analysis of data from MiniBooNE, LSND, KARMEN, NOMAN, Bugey, CHOOZE, CCFR84, and CDHS[9], rather than a model. There is a high probability that light sterile neutrinos with a large mixing angle exist, which is the basis of our work.

There is a strong correlation of the pulsar velocity with temperature, T. Since it is difficult to determine T accurately, it is difficult for us to predict the velocity of a pulsar whose kick arises from sterile neutrino emission. On the other hand, if the pulsar kick arises from the asymmetric emission of active neutrinos produced by the modified URCA processes after 10 seconds, also proportional to $P(0)$ [13], then T can be determined by an accurate measurement of the neutrinos from the supernova. Therefore, in future years, with much more accurate neutrino detectors, one could predict the velocity of the resulting pulsar. Unfortunately, the energy of emitted sterile neutrinos cannot be measured. From our results in the present paper and those in Ref[13], high luminosity pulsars receive a large kick both from sterile neutrinos in the first ten seconds and standard neutrinos in the second ten seconds.

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- [1] A. Aguilar *et al* (LSND Collaboration), Phys. Rev. **D64**, 11207 (2001)
 - [2] M. Sorel, J.M. Conrad, and M.H. Shaevitz, Phys. Rev. **D70**, 073004 (2004)
 - [3] A.A. Aguilar-Arevalo *et al* (MiniBooNE Collaboration), Phys. Rev. Lett. **98**, 231801 (2007)
 - [4] M. Maltoni and T. Schwetz, Phys. Rev. **D76**, 0930005 (2007)
 - [5] M. Sterbenz, LANL, MiniBooNE TN 225
 - [6] M. Maltoni, J. Phys. Conf. Ser. **110**, 082011 (2008)
 - [7] T. Goldman, G.J. Stephenson, Jr. and B.H.J. McKellar, Phys. Rev. **D 75**, 091301 (2007)
 - [8] G.J. Stephenson, T. Goldman, B.H.J. McKellar and M. Garbutt, Int. J. Mod. Phys. **A20**, 6373 (2005)
 - [9] G. Karagiorgi, Z. Djurcic, J.M. Conrad, M.H. Shaevitz, and M. Sorel, arXiv:0906.1997 [hep-ph]
 - [10] L. Wolfenstein, Phys. Rev. **D17**, 2369 (1978); **D20**, 2634 (1979)
 - [11] S.P. Mikheyev and A. Yu Smirnov, Sov. J. Nucl. Phys. **42**, 913 (1985)
 - [12] B.M.S. Hansen and E.S. Phinney, astro-ph/9708071, Mon. Not. R. Astron. Soc. **291**, 569 (1997)
 - [13] E. M. Henley, M. B. Johnson and L. S. Kisslinger, Phys. Rev. **D 76**, 125007 (2007)
 - [14] N.N Chugai, Sov. Astron. Lett. **10**, 87 (1984)
 - [15] O.F. Dorofeev, V. N. Rodionov and I. M. Ternov, Sov. Astron. Lett. **11**, 123 (1985)
 - [16] D. Lai and Y-Z Qian, ApJ. **505**, 844 (1998). This article has references to earlier work on standard neutrinos and pulsar kicks.
 - [17] G. M. Fuller, A. Kusenko, I. Mocioiu and S. Pascoli, Phys. Rev. **D 68**, 103002 (2003)
 - [18] P.L. Bierman and A. Kusenko, Phys. Rev. Lett **96**, 091301 (2006); A. Kusenko, Phys. Rev. Lett **97**, 241301 (2006)
 - [19] A. Kusenko, arXiv:0906.2968 [hep-ph]
 - [20] E.Kh. Akhmedov, M. Maltoni and A. Yu. Smirnov, Phys. Rev. Lett. **95**, 211801 (2005)
 - [21] K. Abazajian, G.M. Fuller and M. Patel, Phys. Rev. **D64**, 023501 (2001)
 - [22] D. Nötzold and G. Raffelt, Nucl. Phys. **B307**, 924 (1988)
 - [23] B.L. Friman and O.V. Maxwell, ApJ **232**, 541 (1979)
 - [24] J.N. Bahcall and R.A. Wolf, Phys. Rev. **140**, B1452 (1965)
 - [25] M.H. Johnson and B.A. Lippman, Phys. Rev. **76**, 828 (1949)
 - [26] J.J. Matese and R.F. O'Connell, Phys. Rev. **180**, 1289 (1969)
 - [27] N. Iwanamoto and C.J. Pethik, Phys. Rev. **D25**, 313 (1982)
 - [28] I. Sagert and J. Schaffner-Bielich, J. Phys. G **35**, 014062 (2008); astro-ph/0708.2352 (2007)
 - [29] A. Burrows and J.M. Lattimer, ApJ. **307**, 178 (1986)
 - [30] A. Burrows, T.J. Mazurek and J.M. Lattimer, ApJ. **251**, 325 (1981)
 - [31] R. Ospanov, Bulletin of APS April Meeting 2008, J2 3